On the Plurality of Initial States

Computational & Operational State Complexity of MFA

Nelma Moreira, Rogério Reis, **Gonçalo Teixeira**

Department of Computer Science
Faculty of Sciences University of Porto

May 7, 2026

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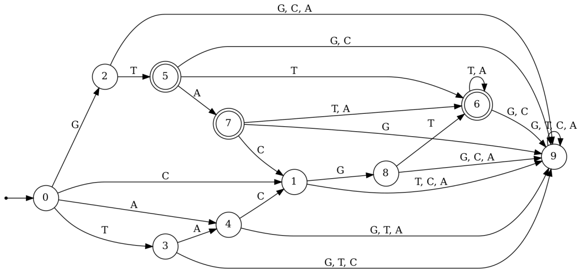
Let's talk about biology

- DNA is essentially a string over the alphabet $\Sigma = \{A, T, G, C\}$
- Certain substrings correspond to certain diseases, mutations, genes, ...
- Sometimes you need to know if people have combinations of them.
- How can we match the following pattern?

$$\mathrm{Suff}(GTACG)T(A+T)^{\star}$$

Deterministically

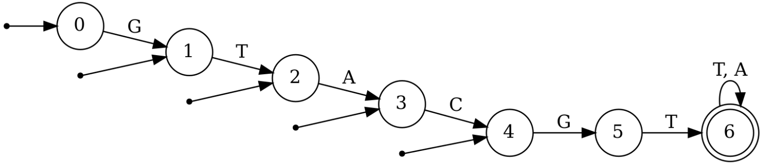
$$\mathrm{Suff}(GTACG)T(A+T)^{\star}$$



- Pro: Easy to implement
- Con: Too big

Nondeterministically

$$\mathrm{Suff}(GTACG)\,T(A + T)^{\star}$$



- Pro: Very small
- Con: Hard to implement

Just Allow More Initial States

- Pro: Small-ish (NFA $\xrightarrow{2^n}$ MFA $\xrightarrow{2^n}$ DFA)
- Pro: Easy to implement
- Con: Not well studied

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MFAs

Definition (MFA)

A Multi-entry Finite Automaton (MFA) is a 5-tuple $M = \langle Q, \Sigma, \delta, I, F \rangle$ where

- Q is a finite set of states
- Σ is a finite alphabet
- $\delta : Q \to \Sigma \to Q$ is a deterministic transition function
- $I \subseteq Q$ is the subset of initial states
- $F \subseteq Q$ is the subset of final states

The language accepted by M is $\mathcal{L}(M) = \{w \in \Sigma^\star \mid \hat{\delta}(I, w) \cap F \neq \emptyset\}$

MFAs

The dual nature of MFAs:

- A DFA extended with multiple initial states
- An NFA with limited nondeterminism

Question

If they are equivalent in terms of computational power, how do they compare in terms of complexity?

MFAs

Operational State Complexity

Given a function $f : (2^{\Sigma^*})^k \to 2^{\Sigma^*}$ and languages $L_1, L_2, \ldots, L_k$, how many states are sufficient and needed to recognize $f(L_1, L_2, \ldots, L_k)$

Computational Complexity

How difficult (space / time) is it to decide if an MFA M has a certain property P

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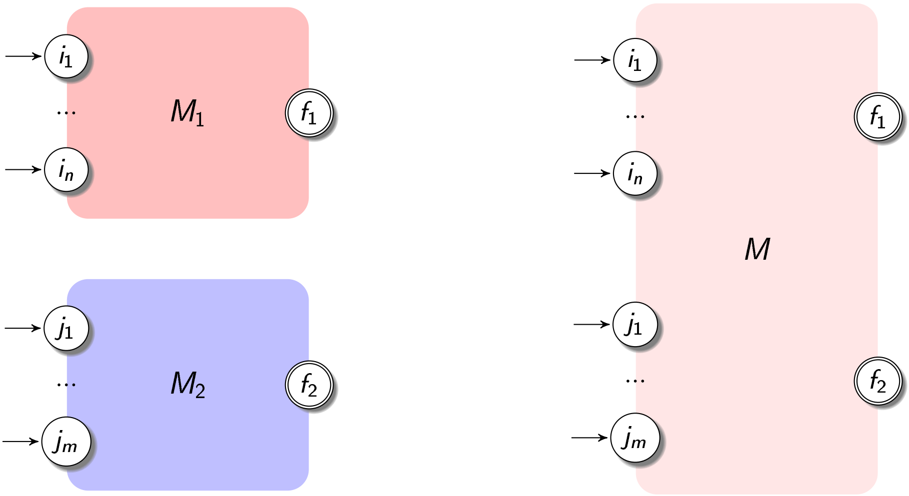
Operational State Complexity

Table: Summary of operational state complexity results.

Operation	DFA	$\|\Sigma\|$	MFA	$\|\Sigma\|$	NFA	$\|\Sigma\|$
$L \cap K$	nm	2	$\mathbf{nm}$	2	nm	2
$L \cup K$	nm	2	$n + m$	2	$n + m$	2
$\Sigma^\star \setminus L$	n	1	2^n	4	2^n	2
$\mathrm{Suff}(L)$	$2^n - 1$	2	n	2	n	2
$\mathrm{Pref}(L)$	n	2	$\mathbf{n}$	2	n	2
$\mathrm{Fact}(L)$	2^{n-1}	2	n	2	n	2
L^R	$2^n - 1$	2	$2^n - 1$	$\mathcal{O}(n^2)$	n	2

Table: ■ previous results ■ good ■ bad

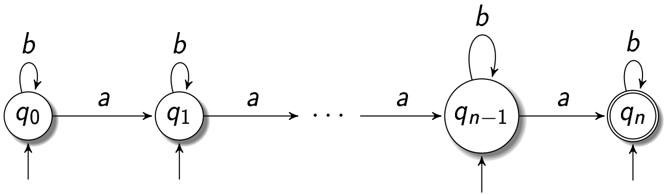
Upper Bound



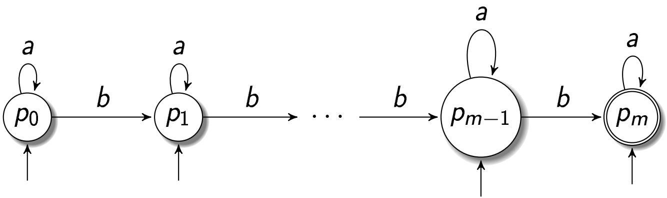
Tightness

To prove tightness we need to present witness languages

- Let $L^a_n = \{w \in \{a,b\}^\star \mid |w|_a \leq n\}$



- Let $L^b_m = \{w \in \{a,b\}^\star \mid |w|_b \leq m\}$



Computational Complexity

Table: Summary of the complexity of decision problems.

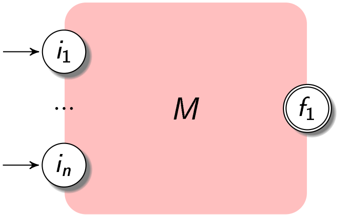
Problem	DFA	MFA	NFA
Membership	L	L	NL-complete
Emptiness	NL-complete	**NL-complete**	NL-complete
Universality	NL-complete	PSpace-complete	PSpace-complete
Inclusion	P	PSpace-complete	PSpace-complete
Minimization	P	PSpace-complete	PSpace-complete

Table: ■ previous results ■ good ■ bad

Membership $\in$ L

Instance: An MFA $M = \langle Q, \Sigma, \delta, I, F \rangle$ and a word $w \in \Sigma^\star$.
Question: Does $w \in \mathcal{L}(M)$?

$$w = \sigma_1 \sigma_2 \cdots \sigma_n$$



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Conclusion

This research clarifies the role that nondeterminism plays in automata based models.

- How does limited nondeterminism behave under standard language-theoretic operations
- How does limited nondeterminism affect complexity-theoretic properties
- Gives theoretical basis to efficient parallel speculative matching algorithms
- Fills a gap in the literature

Our paper "On the Complexity of Multi-entry DFAs" was accepted at DCFS 26 in Canada

Thank You