

On the Plurality of Initial States: Operational State Complexity of Multi-Entry Finite Automata

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1 Background & Aim

Multi-Entry Finite Automata (MFAs) (Gill and Kou, 1974; Holzer et al., 2001; Kappes, 2000) are a generalization of DFAs allowing multiple initial states. This limited form of nondeterminism makes MFAs exceptionally well-suited for high-performance implementation and parallelization, with applications to genomics, linguistics, and network security via speculative evaluation. Despite their recognized utility, the state complexity of language-theoretic operations and the computational complexity of decision problems for MFAs remain largely unexplored. This work aims to bridge this gap by studying the operational and computational complexity of MFAs.

2 Methods

We define the notion of multi-entry state complexity and adapt automata-theoretic tools, such as fooling sets (Birget, 1992; Glaister and Shallit, 1996) and universal witnesses (Brzozowski, 2012), to provide tight lower bounds for MFAs. These tools allow us to evaluate how MFAs behave under standard operations compared to DFAs and NFAs.

3 Results

Our findings show that MFAs strike a balance between being expressive and easy to implement. We prove that MFAs efficiently recognize the intersection, union, and the suffix, prefix, and factor closures of languages. In these cases, MFAs achieve the complexity bounds of NFAs while maintaining a structure far more amenable to parallel execution and often remaining exponentially smaller than their equivalent DFAs. Conversely, we prove that even MFA’s restricted nondeterminism suffices to render the state complexity of the complement exponential and make the decision problems of equality, inclusion, and universality PSPACE-complete.

4 Conclusions

This work contributes to a deeper understanding of the structural and complexity-theoretic properties of MFAs, as well as providing fundamental insights into the nature of nondeterminism and its impact on the hardness of computational problems. Additionally, our work provides insight into which applications could benefit from MFAs as well as serving as a basis for future research into the multi-entry state complexity of unary and sub-regular languages.

Keywords: Automata Theory; Operational State Complexity; Regular Languages; Computational Complexity; Limited Nondeterminism

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References

- Birget, J.-C. (1992). Intersection and union of regular languages and state complexity. *Information Processing Letters*, 43(4), 185–190. [https://doi.org/10.1016/0020-0190\(92\)90198-5](https://doi.org/10.1016/0020-0190(92)90198-5)
- Brzozowski, J. (2012). In search of most complex regular languages. In *Implementation and application of automata* (pp. 5–24). https://doi.org/10.1007/978-3-642-31606-7_2
- Gill, A., & Kou, L. (1974). Multiple-entry finite automata. *Journal of Computer and System Sciences*, 9(1), 1–19. [https://doi.org/10.1016/S0022-0000\(74\)80034-6](https://doi.org/10.1016/S0022-0000(74)80034-6)
- Glaister, I., & Shallit, J. (1996). A lower bound technique for the size of non-deterministic finite automata. *Information Processing Letters*, 59(2), 75–77. [https://doi.org/10.1016/0020-0190\(96\)00095-6](https://doi.org/10.1016/0020-0190(96)00095-6)

- Holzer, M., Salomaa, K., & Yu, S. (2001). On the state complexity of k-entry deterministic finite automata. *Journal of Automata, Languages and Combinatorics*, 6(4), 453–466. <https://doi.org/10.25596/jalc-2001-453>
- Kappes, M. (2000). Descriptive complexity of deterministic finite automata with multiple initial states. *Journal of Automata, Languages and Combinatorics*, 5(3), 269–278. <https://doi.org/10.25596/jalc-2000-269>